Global or Absolute Min/Max

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Differential Equations

Subtopics: Global or Absolute Minima and Maxima, Modelling Situations, Integration Technique - Harder Powers, Accumulation of Change, Total Amount, Fundamental Theorem of Calculus (Second)

Paper: Part B-Non-Calc / Series: 2000 / Difficulty: Medium / Question Number: 4

4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

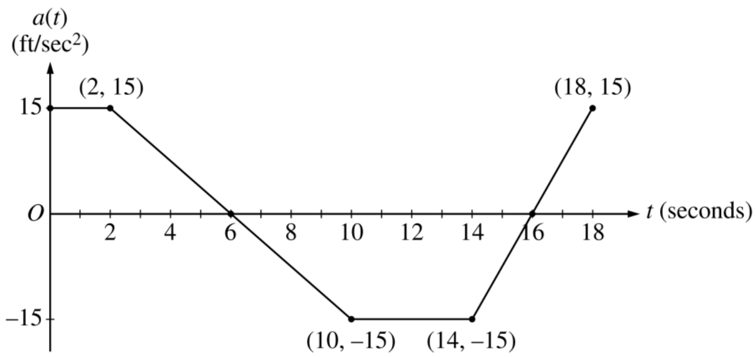
Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Increasing/Decreasing , Integration - Area Under A Curve, Global or Absolute Minima and Maxima, Derivative Graphs

Paper: Part A-Calc / Series: 2001 / Difficulty: Very Hard / Question Number: 3



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.

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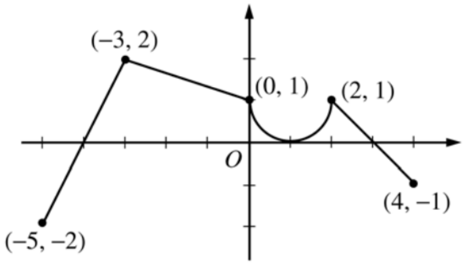
Question 3

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Derivative Graphs, Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Points Of Inflection, Integration Graphs

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Somewhat Challenging / Question Number: 5



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

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Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Accumulation of Change, Total Amount, Increasing/Decreasing , Concavity, Global or Absolute Minima and Maxima

Paper: Part A-Calc / Series: 2004-Form-B / Difficulty: Hard / Question Number: 2

2. For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

   (a) Show that the number of mosquitoes is increasing at time $t = 6$.

   (b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

   (c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

   (d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

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Question 5

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Global or Absolute Minima and Maxima, Differentiation Technique – Chain Rule, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 4

4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

   (a) Find the time t at which the particle is farthest to the left. Justify your answer.

   (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

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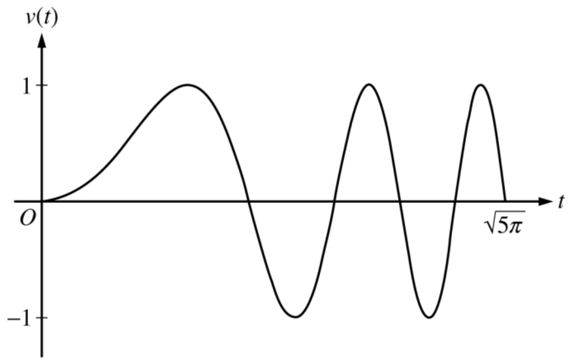
Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Differentiation Technique – Trigonometry, Global or Absolute Minima and Maxima

Paper: Part A-Calc / Series: 2007-Form-B / Difficulty: Hard / Question Number: 2



2. A particle moves along the x-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \le t \le \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

   (a) Find the acceleration of the particle at time $t = 3$.

   (b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

   (c) Find the position of the particle at time $t = 3$.

   (d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

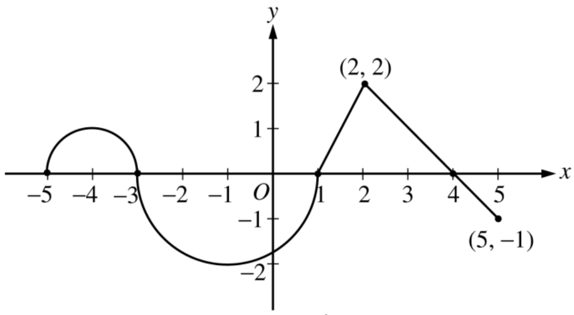
Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Applications of Integration

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Points Of Inflection, Concavity, Increasing/Decreasing , Derivative Graphs, Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Easy / Question Number: 4



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

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Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Rates of Change (Instantaneous), Implicit Differentiation, Modelling Situations, Global or Absolute Minima and Maxima, Related Rates

Paper: Part A-Calc / Series: 2008 / Difficulty: Medium / Question Number: 3

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

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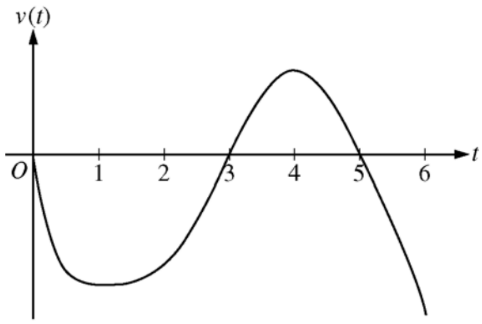
Question 9

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Global or Absolute Minima and Maxima, Intermediate Value Theorem, Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Differentiation Technique – Chain Rule, Differentiation Technique – Trigonometry, Tangents To Curves, Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 4

4. The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4 + t^2}\, dt$ and $g(x) = f(\sin x)$.

(a) Find $f'(x)$ and $g'(x)$.

(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

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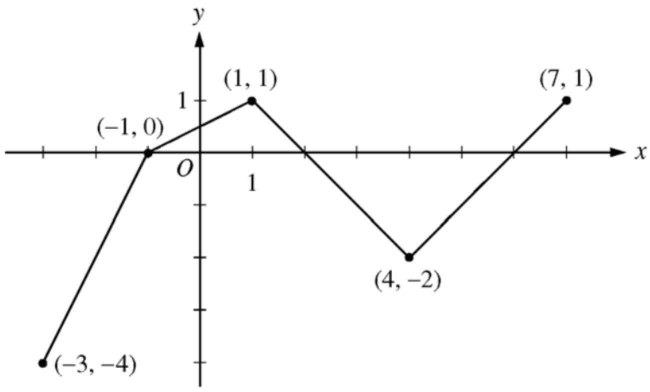
Question 11

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Derivative Graphs, Points Of Inflection, Global or Absolute Minima and Maxima, Rates of Change (Average), Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 5



Graph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

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Question 12

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Modelling Situations, Interpreting Meaning in Applied Contexts, Fundamental Theorem of Calculus (Second), Global or Absolute Minima and Maxima, Local or Relative Minima and Maxima

Paper: Part A-Calc / Series: 2009 / Difficulty: Somewhat Challenging / Question Number: 3

3. Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

   (a) Find Mighty's profit on the sale of a 25-meter cable.

   (b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\, dx$ in the context of this problem.

   (c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

   (d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

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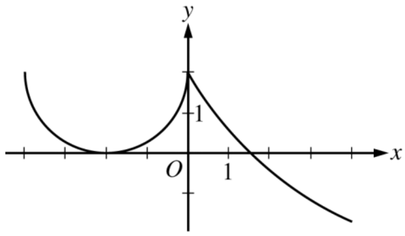
Question 13

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Points Of Inflection, Integration Technique – Geometric Areas, Derivative Graphs, Global or Absolute Minima and Maxima, Differentiation Technique – Exponentials, Integration Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: / Question Number: 6



Graph of f'

6. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}$.

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

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Question 14

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Interpreting Meaning in Applied Contexts, Global or Absolute Minima and Maxima, Modelling Situations

Paper: Part A-Calc / Series: 2009-Form-B / Difficulty: Somewhat Challenging / Question Number: 2

2. A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

   (a) What was the distance between the road and the edge of the water at the end of the storm?

   (b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

   (c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

   (d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

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Question 15

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Rates of Change (Average), Riemann Sums – Trapezoidal Rule, Interpreting Meaning in Applied Contexts, Modelling Situations, Global or Absolute Minima and Maxima

Paper: Part A-Calc / Series: 2010 / Difficulty: Medium / Question Number: 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

   (a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

   (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

   (c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

   (d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

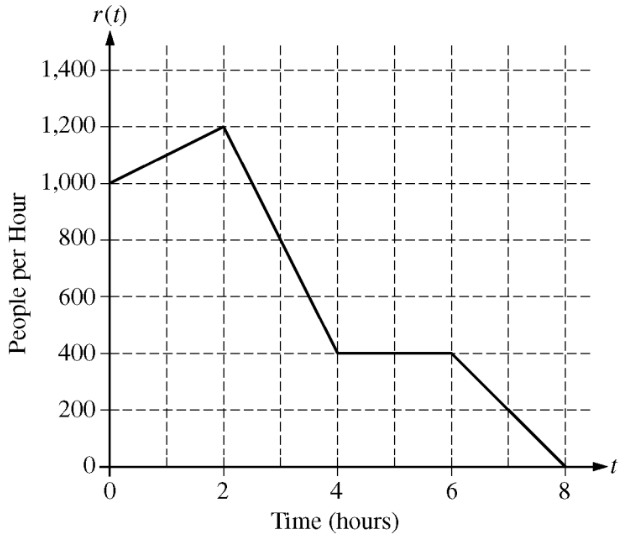
Question 16

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Increasing/Decreasing , Global or Absolute Minima and Maxima, Modelling Situations, Integration Technique – Geometric Areas, Derivative Graphs

Paper: Part A-Calc / Series: 2010 / Difficulty: Easy / Question Number: 3



3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

   (a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

   (b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

   (c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

   (d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

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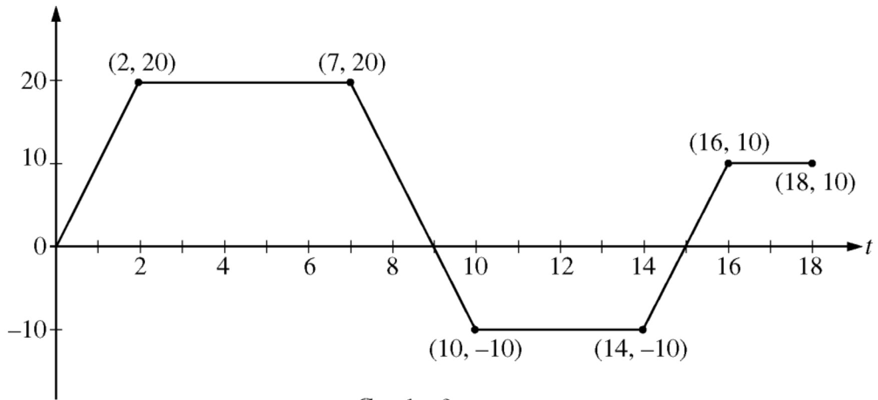
Question 17

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation, Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Global or Absolute Minima and Maxima, Derivative Graphs, Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (Second)

Paper: Part B-Non-Calc / Series: 2010-Form-B / Difficulty: Medium / Question Number: 4



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at that time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

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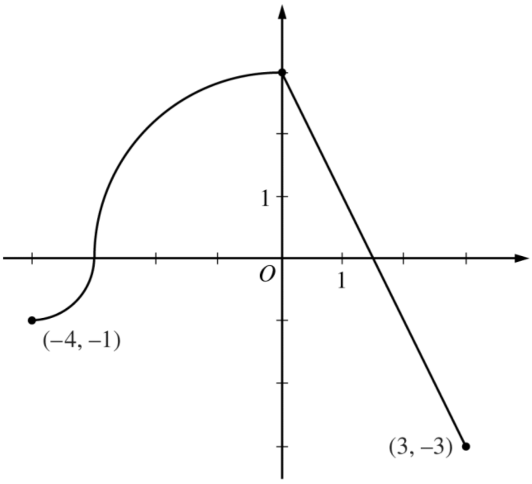
Question 18

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Global or Absolute Minima and Maxima, Points Of Inflection, Rates of Change (Average), Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Easy / Question Number: 4



Graph of f

4. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

   (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

   (b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

   (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

   (d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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Question 19

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Interpreting Meaning in Applied Contexts, Total Amount, Increasing/Decreasing , Global or Absolute Minima and Maxima, Accumulation of Change

Paper: Part A-Calc / Series: 2013 / Difficulty: Easy / Question Number: 1

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

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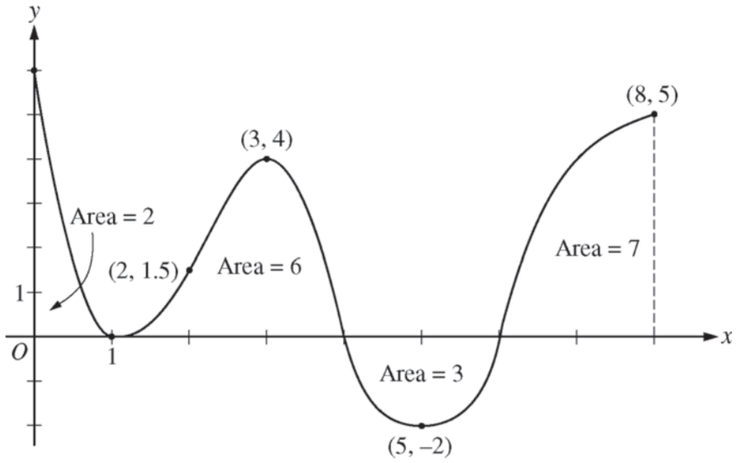
Question 20

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Concavity, Increasing/Decreasing , Implicit Differentiation, Tangents To Curves, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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Question 21

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

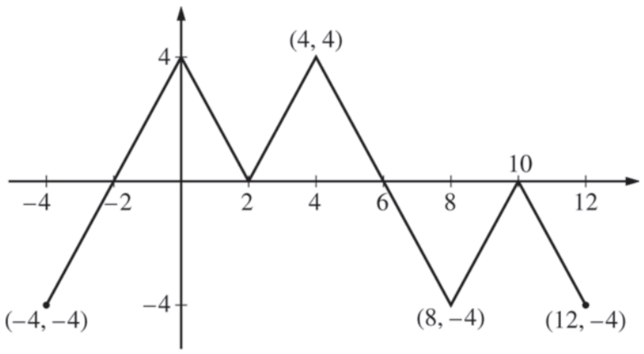
Question 22

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (Second), Points Of Inflection, Global or Absolute Minima and Maxima, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 3



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_2^x f(t)\, dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.

(d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

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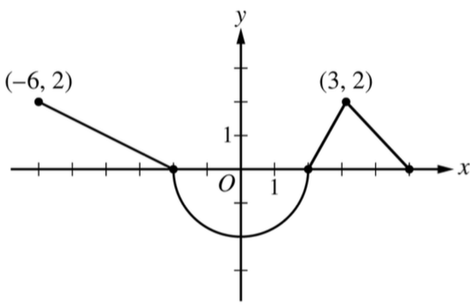
Question 23

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Integration Technique – Geometric Areas, Derivative Graphs, Increasing/Decreasing , Global or Absolute Minima and Maxima, Differentiability

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 3



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find the values of $f(-6)$ and $f(5)$.

   (b) On what intervals is f increasing? Justify your answer.

   (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

   (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

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Question 24

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Global or Absolute Minima and Maxima, Modelling Situations, Accumulation of Change

Paper: Part A-Calc / Series: 2018 / Difficulty: Medium / Question Number: 1

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

Question 25

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Rates of Change (Average), Tangents To Curves, Global or Absolute Minima and Maxima, L'Hôpital's Rule, Calculating Limits Algebraically, Differentiation Technique – Product Rule, Differentiation Technique – Exponentials, Differentiation Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Somewhat Challenging / Question Number: 5

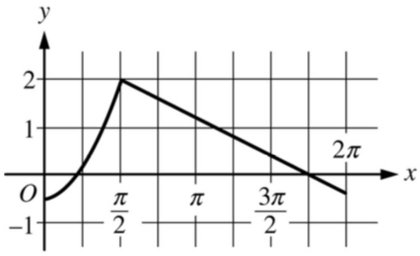
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim\limits_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

Question 26

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Total Amount, Average Value of a Function, Global or Absolute Minima and Maxima, Modelling Situations, Increasing/Decreasing , Accumulation of Change

Paper: Part A-Calc / Series: 2019 / Difficulty: Somewhat Challenging / Question Number: 1

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?

(c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$) ? Explain your reasoning.

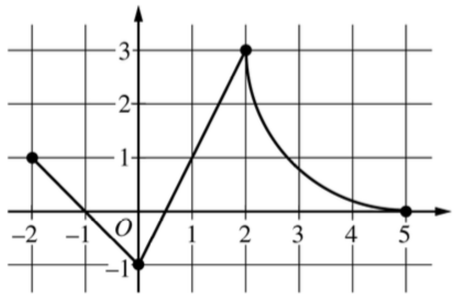
Question 27

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (First), Global or Absolute Minima and Maxima, Calculating Limits Algebraically, Integration Graphs

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 3



Graph of f

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3-\sqrt{5})$ is on the graph of f.

(a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

(b) Evaluate $\int_{3}^{5} (2f'(x) + 4)\,dx$.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

(d) Find $\lim_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

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Question 28

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Total Amount, Average Value of a Function, Increasing/Decreasing , Global or Absolute Minima and Maxima, Fundamental Theorem of Calculus (Second), Accumulation of Change

Paper: Part A-Calc / Series: 2022 / Difficulty: Easy / Question Number: 1

1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

   (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

   (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\,dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

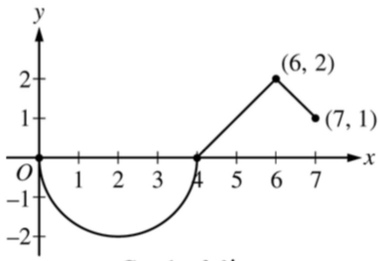
Question 29

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Derivative Graphs, Integration Technique – Geometric Areas, Points Of Inflection, Increasing/Decreasing , Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Easy / Question Number: 3



Graph of f'

3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

(a) Find $f(0)$ and $f(5)$.

(b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

(c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

(d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

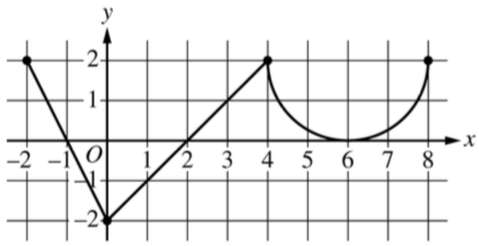
Question 30

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Concavity, Derivative Graphs, Global or Absolute Minima and Maxima, Integration Technique – Geometric Areas, L'Hôpital's Rule, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Medium / Question Number: 4



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

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